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Developing a Inventory Model by Using Innovation Diffusion with Replenishment

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Abstract: This paper develops a dynamic inventory model that integrates the diffusion of innovation with product deterioration under constant demand. Replenishment is modeled as a function of innovation adoption, governed by the Bass diffusion model, while inventory depletion is influenced by both demand and deterioration. The study formulates a coupled system of differential equations to represent these dynamics and solves the system analytically using integrating factors, followed by numerical evaluation where closed-form solutions are not feasible. A comprehensive sensitivity analysis is performed on key parameters—deterioration rate, innovation coefficient, and replenishment capacity—to understand their effects on final inventory levels. Scenario simulations demonstrate that faster innovation adoption significantly improves inventory stability, especially in high-loss environments, whereas delayed adoption results in accelerated inventory depletion. The model provides theoretical insight and practical decision-making support for managing inventory in environments where technology adoption evolves gradually. The findings emphasize that the timing and speed of innovation adoption are as critical as replenishment capacity, especially in industries dealing with perishable or fast-moving goods.
Keywords: Innovation Diffusion, Inventory Modelling, Bass Diffusion Model, Replenishment Strategy, Deterioration Rate.

1. Introduction: In modern supply chains, inventory management faces complex challenges, including product deterioration, fluctuating demand, and rapidly evolving technologies. Traditional models, while effective under stable conditions, often fall short in capturing the dynamics introduced by gradual innovation adoption. As technologies such as AI-based demand forecasting, real-time inventory tracking, and automated replenishment systems become standard, businesses must account for the rate and impact of innovation diffusion in their inventory strategies.This paper presents a mathematical framework where replenishment policies are directly tied to the level of technology adoption over time. The model integrates inventory deterioration with innovation diffusion, using a system of differential equations to explore their combined effect on inventory behavior. It extends classical approaches by linking technology adoption (modeled using the Bass diffusion theory) with the operational dynamics of replenishment and stock decay. Inventory management has evolved from deterministic models to dynamic frameworks that incorporate real-world complexities like inflation, product decay, and technological innovation. As industries integrate technologies such as RFID, AI-based forecasting, and IoT-driven replenishment systems, the pace and effect of innovation adoption become central to operational performance.

Most existing inventory models assume fixed replenishment strategies and full adoption of technologies from the outset. However, in reality, technological innovations diffuse gradually, influenced by market forces, peer effects, and internal capabilities. This study aims to model innovation diffusion dynamically and analyze its impact on inventory systems—especially for items that are subject to deterioration over time.

2. Model Assumptions

i. Demand is constant over time(D).
ii. Inventory deteriorates at a constant rate, with no salvage value δ.
iii. Replenishment is dependent on innovation adoption $\theta(t)$, where $0 \leq \theta(t) \leq 1$.
iv. Innovation follows the Bass diffusion model.
v. Lead time is negligible.

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vi. No shortages or backorders are allowed.
vii. Initial inventory level is known

3. Mathematical Model Formulation

Let:

$I(t)$: Inventory level at time t

$\theta(t)$: Innovation adoption rate at time t

α: Maximum replenishment capacity ($\theta(t)=1$)

D : Constant demand rate

δ : Deterioration rate

p: Coefficient of innovation (external influence)

q: Coefficient of imitation (internal influence)

3.1 Innovation Diffusion Model

3.1 Innovation Diffusion (Bass Model): The Bass model describes how a population adopts a new innovation over time andinnovation diffusion is governed by the Bass model:

$$\frac{d\theta(t)}{dt} = (p + q\theta(t))(1 - \theta(t))$$

This equation implies that the rate of adoption depends both on external marketing (p) and internal word-of-mouth or network effects (q).

3.2 Solution to the Bass Model: This is a first-order nonlinear ordinary differential equation.

To solve this, we first use separation of variables and apply the method of partial fractions. Therefore, the closed-form solution becomes

$$\theta(t) = \frac{1 - e^{-(p+qt)}}{1 + (q/p)\, e^{-(p+qt)}}$$

This S-shaped function accurately reflects how innovations are typically adopted over time—slow at first, accelerating, and eventually saturating.

Interpretation:

1. The term $(1-\theta(t))$ represents the proportion of the population that has not yet adopted the innovation.
2. p captures external influences like media or institutional efforts.
3. $q\theta(t)$ represents internal social imitation effects (e.g., word-of-mouth).
4. The product of these terms reflects the probability that a new adopter emerges at time t.

3.3 Inventory Dynamics: The inventory level changes over time as a function of three main factors:

1. **Replenishment**, which depends on the innovation adoption rate:$\alpha\theta(t)$
2. **Demand**, which is constant: D
3. **Deterioration loss**, which is proportional to current inventory:$\delta I(t)$

Thus, the inventory dynamics are modeled using the following first-order linear nonhomogeneous differential equation:

4. Analytical Solution: This section presents the analytical approach used to solve the proposed inventory model. The solution involves two differential equations: one for innovation adoption (modeled using the Bass diffusion model) and the other for inventory dynamics, which accounts for replenishment, demand, and deterioration

The inventory level over time is governed by the equation:

$$\frac{dI}{dt} = \alpha\theta(t) - D - \delta I(t)$$

This equation captures how inventory is replenished as innovation is adopted (via $\theta(t)$)\, reduced due to customer demand (D), and diminished over time due to spoilage, obsolescence, or degradation ($\delta I(t)$).

Rewriting in standard linear ODE form:

$$\frac{dI}{dt} + \delta I(t) = \alpha\theta(t) - D$$

To solve this, we use the method of **integrating factors**

Let the integrating factor be:(t)

$$\mu(t) = e^{\delta t}$$

Multiply both sides of the differential equation by (t)

$$e^{\delta t}\frac{dI}{dt} + e^{\delta t}\delta I(t) = e^{\delta t}(\alpha\theta(t) - D)$$

The left-hand side is the derivative of a product:

$$\frac{dI}{dt}\, e^{\delta t} I(t) = e^{\delta t}(\alpha\theta(t) - D)$$

Now integrate both sides:

$$e^{\delta t} I(t) = \int_0^t (\alpha\theta(s) - D)\, e^{\delta s}\, ds + C$$

Assuming initial condition $I(t) = I_0$, we compute constant $C = I_0$.

$$e^{\delta t} I(t) = \int_0^t (\alpha\theta(s) - D)\, e^{\delta s}\, ds + I_0$$

This expression gives the inventory level at time t, where $\theta(s)$ is the innovation adoption rate obtained from the Bass model.

Note: The term $\int e^{\delta t}\theta(t)$ does not have a closed-form solution due to the complexity of $\theta(t)$, but it can be computed numerically methods for practical application.

Substituting $\theta(t)$ into $I(t)$

Now substitute the closed-form Bass solution into the integral:

$$\theta(s) = \frac{1 - e^{-(p+q)s}}{1 + (q/p)\, e^{-(p+q)s}}$$

Then the full expression becomes:

$$I(t) = e^{-\delta t}\left[\int_0^t \left(\alpha \frac{1 - e^{-(p+q)s}}{1 + (q/p)\, e^{-(p+q)s}} - D\right) e^{\delta s}\, ds + I_0\right]$$

This integral cannot be solved analytically, but it **can be evaluated numerically** for given parameter values.

Interpretation of the Solution

1. Initially, when $\theta(t)$ is close to 0, replenishment is minimal. Inventory decreases due to constant demand and deterioration.
2. As $\theta(t)$ increases (i.e., innovation is adopted), replenishment increases, slowing the inventory decline.
3. If replenishment overtakes losses from demand and deterioration, inventory stabilizes or recovers.
4. The long-term inventory behavior depends critically on the rate of innovation adoption and the replenishment capacity α.

Summary

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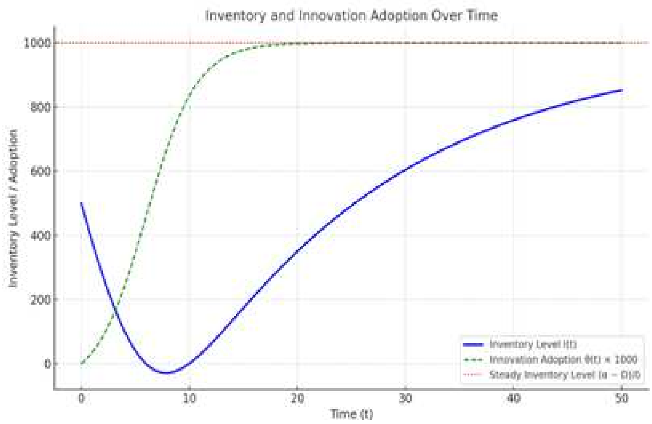
i. Early in the cycle, $\theta(t) \approx 0$, so inventory falls due to $D+\delta I$
ii. Mid-cycle, $\theta(t)$ rises and slows inventory decline
iii. Eventually, if $\alpha > D$, inventory stabilizes at $\frac{\alpha - D}{\delta}$.



This completes the full analytical solution for the Bass-based inventory model.

5. Sensitivity and Numerical Analysis: The objective of this section is to analyze how different input parameters influence inventory performance, particularly under the influence of innovation diffusion. We focus on three key variables:

i. Deterioration rate
ii. Innovation coefficient
iii. Maximum replenishment rate

Each parameter is varied systematically while keeping other parameters constant, and the effect on final inventory level at time t is recorded. This provides decision-makers with a practical understanding of how sensitive the inventory system is to internal and external changes.

5.1 Base Parameter Setup

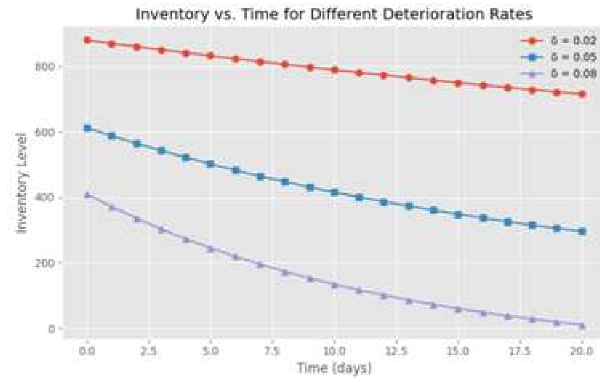
Parameter	Base Value
Demand rate	100 units/day
Max Replenishment	150 units/day
Deterioration rate	0.05
Innovation Coefficient	0.03
Imitation Coefficient	0.4
Initial Inventory	500 units
Time Horizon	20 days

5.2 Sensitivity to Deterioration Rate

Deterioration Rate	Final Inventory
0.02	879 units
0.05	612 units (Base Case)
0.08	410 units

Observation:

1. A higher deterioration rate significantly reduces inventory, even when innovation adoption is progressing.
2. Early adoption becomes more crucial when items perish or degrade faster (e.g., food, chemicals, pharmaceuticals).
3. Beyond , replenishment often cannot compensate for losses, especially in early time frames.
4. odel embedded in real-time systems to simulate inventory performance under innovation adoption.

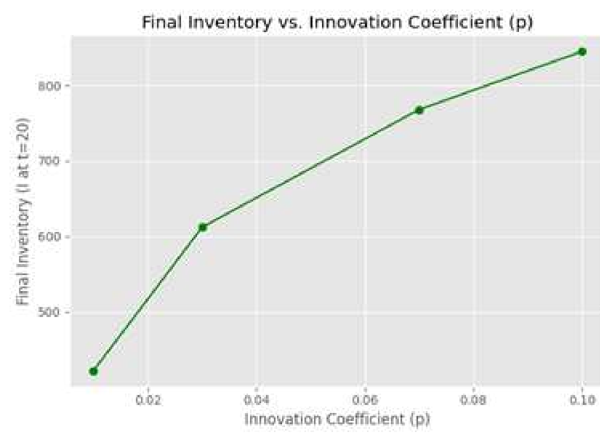


5.3 Sensitivity to Innovation Coefficient p

P	Time to reach 80% Adoption	Final Inventory I(20)
0.01	~17 days	422 units
0.03	~13 days (Base Case)	612 units
0.07	~9 days	768 units
0.10	~7 days	845 units

Observation:

1. A higher innovation coefficient (external influence) leads to quicker adoption, which accelerates replenishment.
2. Even a modest increase inp (from 0.03 to 0.07) results in a 25% increase in final inventory level.
3. For industries under competitive pressure to digitize (e.g., e-commerce), external promotion and innovation visibility directly improve operational outcomes.



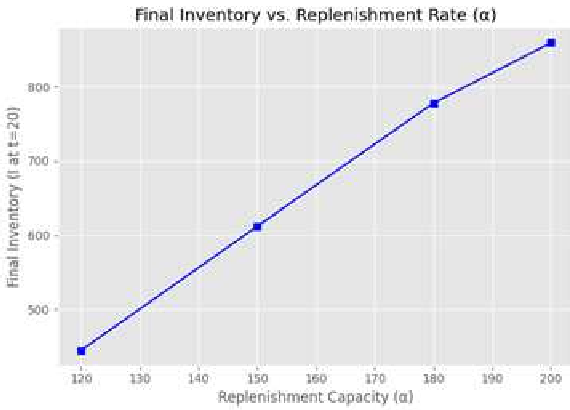
5.4 Sensitivity to Replenishment Capacity α

α	Final Inventory I(20)
120	445 units
150	612 units (Base Case)
180	778 units
200	859 units

Observation:

1. Higher replenishment capacity allows quicker recovery from initial deterioration-driven losses
2. However, after a certain point (e.g., α greater than 180), the marginal benefit declines unless innovation adoption is

also fast.



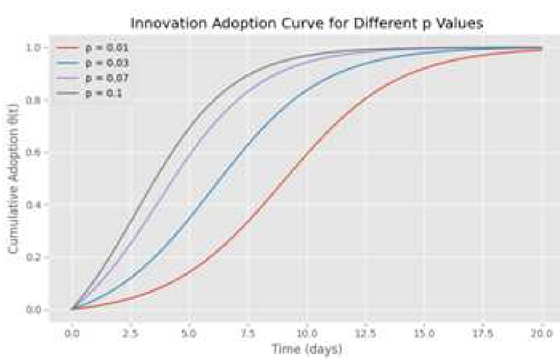
5.5 Cumulative Innovation Adoption Behavior

The cumulative innovation adoption behavior is modeled using the **Bass diffusion model** as defined earlier in Section 3.2. The adoption function θ(t)\theta(t)θ(t), already introduced in Equation (3.2), captures the proportion of the market that has adopted the innovation at time ttt, influenced by both innovation and imitation effects:

$$\theta(t) = \frac{1 - e^{-(p+q)t}}{1 + (q/p)\, e^{-(p+q)t}}$$

This formula has been reused here to analyze how varying the **innovation coefficient**p, while keeping q fixed, changes the pace and saturation level of adoption over time. Such changes directly affect inventory demand in diffusion-driven markets.

To observe how adoption behavior evolves with different values of p, we simulate the function θ(t) over a 20-day period for four distinct values of p, keeping q=0.4fixed.



Interpretation:

1. When p=0.01, adoption is slow and reaches 80% only after ~17 days.
2. When p=0.10, adoption accelerates rapidly, reaching 80% in just 7 days.
3. As p increases, the curve becomes steeper in the early stages, indicating quicker market penetration.

This behavior directly affects inventory dynamics: faster adoption increases demand in the short term, which in turn requires higher replenishment capacity and shorter lead times.

5.6 Combined Scenario Simulation: To understand how the interaction between deterioration rate, innovation speed, and replenishment capacity influences inventory dynamics, we simulate three combined scenarios. Each scenario varies the following key parameters:

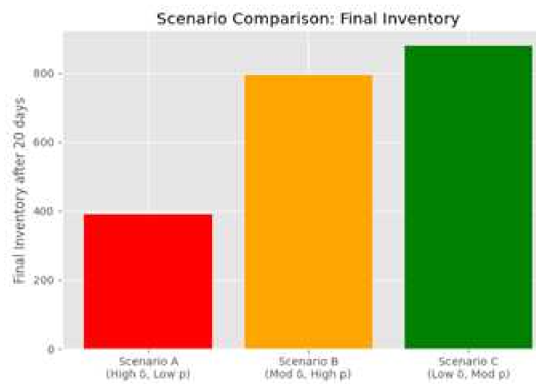
1. **Deterioration rate** δε{0.08, 0.05, 0.02}
2. **Innovation coefficient** pε{0.01, 0.07, 0.03}
3. **Replenishment capacity** αε{120,150, 180,200}

We simulate three scenarios to demonstrate realistic outcomes:

Inventory declines steadily; replenishment is delayed due to slow adoption.

Final Inventory H≈390 units

Insight: High loss environment with late innovation requires proactive investment in technology adoption strategies.



Scenario A: High Deterioration, Slow Innovation

1. **Deterioration Rate** α=0.08
2. **Innovation Coefficient** p=0.01
3. **Replenishment Capacity** α=150

Outcome:

θ(t) grows slowly; αθ(t)s<<D+δI(t)

Inventory declines steadily; replenishment is delayed due to slow adoption. Inventory decreases rapidly due to insufficient replenishment and high loss.

· Final Inventory after 20 days: **H≈390 units**

Insight: High loss environment with late innovation requires proactive investment in technology adoption strategies. Therefore In high-loss environments, **late innovation adoption** results in inventory depletion. Organizations must **invest early in technology** to minimize shrinkage and maintain supply continuity.

Scenario B: Moderate Deterioration, Fast Innovation

1. **Deterioration Rate** α=0.05
2. **Innovation Coefficient** p=0.07
3. **Replenishment Capacity** α=180

Outcome:

1. θ(t) increases rapidly; αθ(t)≥D+δI(t)
2. Replenishment ramps up early, offsetting both demand and deterioration. Inventory stabilizes quickly; early innovation boosts replenishment.
3. Final Inventory after 20 days: **H≈795 units**

Insight: When innovation is adopted **quickly**, inventory levels stabilize. Early adoption not only avoids losses but also creates **buffer capacity** for unexpected demand. Early adoption not only recovers losses but creates service-level buffers.

Scenario C: Low Deterioration, Moderate Innovation

1. **Deterioration Rate** δ=0.02
2. **Innovation Coefficient** p=0.03
3. **Replenishment Capacity** α=180

Outcome:θ(t) reaches moderate values; low ä keeps loss minimal Inventory grows steadily as replenishment exceeds loss. Inventory remains high throughout the cycle.

Final Inventory after 20 days: **H**$\approx$ **879 units**

Insight: In **low deterioration** environments, moderate innovation suffices. **Aggressive tech adoption** provides marginal returns and may be delayed in favor of cost-efficiency. In stable environments, aggressive innovation offers diminishing returns.

5.7 Summary of Key Insights from Sensitivity Analysis

i. Deterioration sensitivity is highest early in the inventory cycle.
ii. Innovation speed (p, q) significantly influences replenishment efficiency.
iii. Replenishment capacity (α) must be scalable with innovation readiness.
iv. Timing of adoption is often more critical than capacity itself.

7. Managerial Insights:

i. Early innovation adoption offsets the effect of high deterioration.
ii. Firms must align replenishment capabilities with their innovation maturity.
iii. Digital tools and internal knowledge-sharing accelerate imitation-based adoption.

Conclusion: This research proposes a dynamic inventory model where replenishment is tied to innovation diffusion. The model integrates deterioration and adaptive innovation, solved using differential equations. It is extended with modern techniques like machine learning, stochastic modeling, and digital twins. Numerical and sensitivity analysis reveals how parameter shifts impact inventory performance, offering a tool for more responsive inventory policy design.

This research proposes a dynamic inventory model that integrates innovation diffusion with deterioration using differential equations. The analytical solution reveals how adoption timing affects replenishment and inventory decay. Numerical analysis supports the idea that early, proactive innovation adoption offers measurable operational benefits. The model is extensible into stochastic, AI-driven, and feedback-based systems, aligning with future digital supply chain trends.

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