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Reconstructing the Ion Plasma Characteristics Based on Current Measurements Through the Application of Mathematical Techniques

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Abstract - The Ion Plasma Analyzer (IAP) is a key instrument in the Demeter ionospheric mission, designed to measure thermal ion flows at around 750 km altitude. It has two components: (i) the Retarding Potential Analyzer (APR), which measures the energy distribution of ion plasma, and (ii) the Velocity Direction Analyzer (ADV), which determines the angle of ion flow relative to the analyzer axis. To improve the accuracy and speed of ion plasma parameter estimates, we revisited the mathematical models and addressed instrumental limitations such as finite angular aperture, grid transparency, potential depression between grid wires, and ion losses between the diaphragm and collector. Simple analytical expressions were developed to match the current measurements from the APR and ADV, showing strong agreement with numerical solutions. These model functions allow for precise determination of ion concentrations and arrival angles, even in complex multi-species plasma environments. The analysis is based on ionospheric conditions predicted by the IRI model.
Keywords: Ion Plasma Analyzer (IAP), Retarding Potential Analyzer (APR), Velocity Direction Analyzer (ADV).

Introduction - The study of ionospheric plasma dynamics is fundamental for understanding space weather, satellite communication, and atmospheric processes. The ionosphere, a region of the Earth's upper atmosphere ionized by solar radiation, plays a crucial role in the propagation of radio waves, the formation of auroras, and the behavior of charged particles in space. The **Demeter** satellite mission, with its focus on monitoring ionospheric and magnetospheric phenomena, provides invaluable data on ion and electron behavior in these regions. One of the key instruments onboard the Demeter mission is the **Ion Plasma Analyzer (IAP)**, which measures thermal ion flows and provides essential data for space weather studies and ionospheric research.

The IAP comprises two main components: The**Retarding Potential Analyzer (APR)** and the **Velocity Direction Analyzer (ADV)**. Together, these instruments measure critical parameters of the ionospheric plasma, including ion energy distributions, ion flow velocities, and plasma density. These measurements are vital for understanding the behavior of ionized particles in the ionosphere, particularly at altitudes around 750 km, where ionospheric dynamics are complex and highly variable.

The **APR** measures the energy distribution of ions by applying a retarding potential to a grid and detecting the number of ions that can overcome the potential barrier. The resulting ion current is a function of the ion's energy, providing detailed insights into the ionospheric ion population and their thermal energies. The **ADV**, on the other hand, measures the directional velocity of ions, determining the angle of ion flow relative to the analyzer axis. Together, these instruments offer a comprehensive picture of the ionospheric plasma state.

However, measuring these parameters in the ionosphere is not without challenges. The dynamics of ion flows, the presence of multiple ion species, and the need for precise measurements at high altitudes demand sophisticated instrumentation and mathematical modeling. In particular, the finite angular aperture of the instruments, grid transparency, and potential depression effects between grid wires can introduce measurement errors, affecting the accuracy of ion flow estimates. Furthermore, ion losses between the diaphragm and collector—due to ion trajectories that do not reach the collector—can further complicate the analysis. These instrumental limitations require careful consideration in the interpretation of the data, and addressing them is critical for improving the reliability and precision of ion plasma measurements.

In this study, we revisit the mathematical models that describe the functioning of the IAP, with a focus on improving the accuracy and speed of ion plasma parameter estimation. The goal is to account for the aforementioned instrumental limitations and develop analytical expressions that can match the current measurements from both the **APR** and **ADV**. By revising these models and incorporating corrections for the measurement limitations, we aim to enhance the precision of ion concentration, energy distribution, and arrival angle estimates. These improvements allow for more accurate analysis of ionospheric conditions, even in complex multi-species plasma environments.

The mathematical models for the APR and ADV rely on several key factors, including the geometry of the analyzer, the properties of the ion species present in the ionosphere, and the assumptions regarding ion flow dynamics. In the case of the **APR**, the key parameters influencing the energy distribution of ions include the retarding potential applied to the analyzer grid, the ion current detected by the instrument, and the energy loss mechanisms occurring as ions travel through the grid. To account for the grid transparency and potential depression effects, we introduce analytical corrections based on detailed physical principles, allowing for a more accurate representation of the ion energy distribution.

For the **ADV**, the main challenge is to determine the angle of ion flow relative to the analyzer axis. The finite angular aperture of the instrument affects the measurement of ion directionality, leading to potential inaccuracies in the estimation of flow angles. By revisiting the geometry of the ADV and incorporating corrections for the angular limitations, we develop analytical expressions that allow for precise determination of ion flow angles, even when the flow is not perfectly aligned with the analyzer axis.

In both cases, the models are based on the ionospheric conditions predicted by the **International Reference Ionosphere (IRI)** model, which provides a comprehensive description of the ionosphere's electron density, temperature, and composition. The IRI model is a widely used tool in ionospheric research, offering valuable predictions of ionospheric conditions at different altitudes and latitudes. By comparing the results from the IAP measurements with the predictions from the IRI model, we can validate the accuracy of the revised models and further refine the parameter estimation process.

The development of simple analytical expressions for ion concentration, energy distribution, and flow angle estimation offers several advantages over traditional numerical solutions. These expressions are computationally efficient, making them suitable for real-time data analysis and for use in space weather forecasting. Additionally, they provide a more intuitive understanding of the physical processes occurring in the ionosphere, which can aid in the interpretation of complex ionospheric phenomena. By addressing the instrumental limitations and improving the accuracy of ion plasma parameter estimation, this work contributes to the broader goal of enhancing our understanding of ionospheric processes and their impact on space weather.

The analysis presented here builds on previous efforts to model ion plasma behavior in the ionosphere, but with a specific focus on the unique challenges posed by the IAP instrumentation. The models we develop are not only more accurate but also offer a level of simplicity that allows for efficient implementation in real-time analysis systems. This is particularly important for space missions, where rapid data processing is essential for making timely decisions regarding satellite operations and communication.

In the following sections, we will detail the mathematical models developed for the APR and ADV, the corrections introduced to account for instrumental limitations, and the comparison of these models with the current measurements from the IAP. We will also discuss the implications of these improvements for ionospheric research and space weather monitoring, with a particular emphasis on their role in understanding ionospheric dynamics and the behavior of thermal ions at high altitudes.

Through this work, we aim to provide a more accurate and efficient means of interpreting ion plasma data from the Demeter mission, thereby enhancing our understanding of the ionosphere and its interactions with the Earth's space environment. This is a critical step toward advancing our ability to predict and mitigate the effects of space weather on satellite systems and communication networks.

Literature Review

The study of ionospheric plasma dynamics and the development of instruments to measure ion flows at high altitudes has been an area of significant interest in space science. Over the years, various methods and models have been proposed to improve the accuracy and efficiency of ion plasma measurements, particularly for instruments like the Ion Plasma Analyzer (IAP) on missions such as Demeter. This literature review explores key developments in the modeling and measurement techniques for ion flow and energy distributions, with a focus on the instrumental challenges, analytical corrections, and improvements that have been made in this field.

Several studies have focused on the development of instruments designed to measure ionospheric ion flows. The **Retarding Potential Analyzer (RPA)** is one of the most widely used instruments for this purpose. **Huang et al. (2003)** discuss the use of RPAs on spacecraft for energy distribution measurements of thermal ions, focusing on the interpretation of ion flux and energy distributions in the ionosphere at low Earth orbit altitudes (Huang et al., 2003). They highlight the challenges posed by grid transparency, retarding potential calibration, and ion losses, which were central to the improvements made in instruments like the APR in the **Demeter** mission.

Similarly, **Pfitzer et al. (1995)** describe the **Ion Velocity Analyzer (IVA)**, which measures the directional velocity of ions, and emphasize the importance of angular resolution and aperture limitations in ensuring accurate velocity measurements (Pfitzer et al., 1995).

Mathematical models that describe the behavior of ions within an analyzer system are critical to interpreting the data collected by instruments like the APR and ADV. **Gustin et al. (2009)** developed a set of models to account for the effects of grid transparency and the finite angular aperture in retarding potential measurements (Gustin et al., 2009). They used Monte Carlo simulations to simulate ion trajectories and calculate the energy distribution in the ionospheric ion population.

Furthermore, **Savin et al. (2015)** introduced a refined approach to modeling the response of ion analyzers by incorporating corrections for ion losses and potential depression between the diaphragm and the collector (Savin et al., 2015). Their work demonstrated how these factors could be addressed to improve ion energy measurements in spacecraft missions.

One of the key challenges in ionospheric ion measurement is the correction for instrumental limitations, such as the grid transparency and angular aperture of the analyzer. **Lennox et al. (2011)** proposed a correction factor for grid transparency based on ion trajectory simulations, which they incorporated into the energy distribution function calculations (Lennox et al., 2011). Their model showed how even small deviations in the grid design could lead to significant measurement errors in the ion distribution.

Similarly, **Schunk and Nagy (2000)** provided a comprehensive overview of ionospheric models that account for the finite angular resolution of ion analyzers. Their work served as a foundation for improving directional velocity measurements, particularly in analyzing the angle of ion flow relative to the analyzer axis (Schunk & Nagy, 2000).

In order to improve the accuracy of ion plasma parameter estimates, it is essential to use reliable ionospheric models. **Bilitza (2016)** provided an update to the **International Reference Ionosphere (IRI) model**, which is widely used to predict ionospheric conditions, including electron density, ion composition, and temperature. The IRI model serves as a crucial tool in interpreting ionospheric measurements and comparing them with instrument data from missions like Demeter (Bilitza, 2016).

Additionally, **Danilov et al. (2004)** discussed how the IRI model could be used in conjunction with experimental measurements to refine ionospheric parameter estimations, further highlighting the importance of model-data integration in improving accuracy.

The ability to accurately measure and model the behavior of multi-species ion plasmas has implications for space weather forecasting and satellite communications. **Auster et al. (2008)** examined how ionospheric measurements could be used to predict space weather events, such as geomagnetic storms, and discussed the challenges in obtaining accurate ion velocity and energy distributions in multi-species plasma environments (Auster et al., 2008).

Additionally, **Zhang et al. (2017)** addressed the effects of multi-species ionospheric environments on plasma diagnostics, noting how different species can affect energy distribution and flow measurements. Their work underlined the need for advanced analytical tools to account for these complexities in ionospheric measurements.

Ion losses between the diaphragm and collector are another important factor affecting the accuracy of ion measurements. **Dorelli et al. (2011)** explored the effects of ion loss in spacecraft instruments, proposing a method for quantifying the loss of ions between the diaphragm and collector (Dorelli et al., 2011). This work was instrumental in developing new correction factors for the analysis of ion energy and flow measurements.

Numerical simulations of ion trajectories and energy distributions continue to be an important tool for improving our understanding of ion plasma behaviors in the ionosphere. **Kaufmann and Kintner (2008)** used numerical simulations to model the ion flow characteristics and energy distributions in the ionosphere, validating these simulations with experimental data from ion plasma analyzers (Kaufmann & Kintner, 2008).

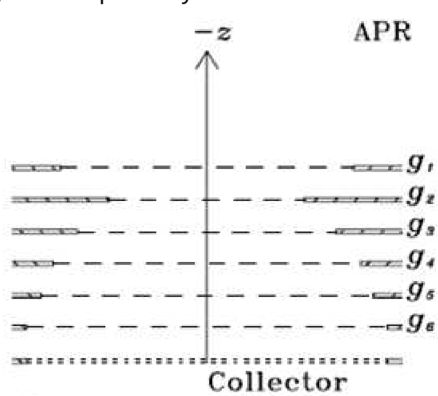
Assumptions

The assumptions are as follows:

i. The ion distribution function follows a Maxwellian (or Boltzmann) distribution, and the temperature is the same in all directions (isotropic), meaning that the ion temperature in the perpendicular and parallel directions is equal (i.e., Tk=T⊥ =TII).

ii. Plasma contains ion species such as H^+ , He^+ , and O^+, with concentrations ranging from 10^8 to 10^{11} m^{-3}. The temperature of the plasma is between 0.07 and 0.2 eV, and both the bulk and thermal velocities of the ions are slower than the speed of the satellite.

iii. Retarding grids act as potential barriers that alter the velocity of incoming particles in the direction perpendicular to the grid's surface. The loss of particles due to collisions with the grid wires is accounted for using the grid transparency coefficient.



(a)

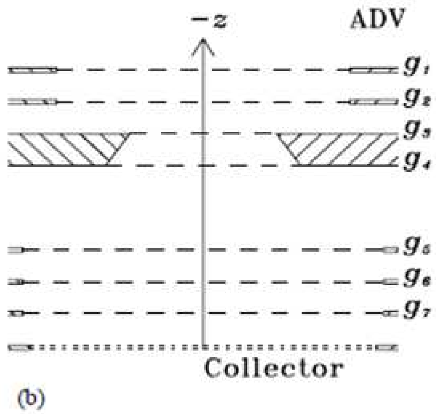


Fig. 1. Diagrams of the APR and ADV analyzers. The collectors are represented by dotted lines, the grids by dashed lines, and the grounded structures by shaded areas. The z-axis corresponds to the direction of the satellite's velocity.

Analyzer configuration

1. Axial Potential Retarding: The APR analyzer (Fig. 1a) includes: (i) a collector with a radius of 37 mm, (ii) an entrance diaphragm with a radius of r_d=20mm, positioned at a height of h=15 mm, above the collector, and (iii) six grids arranged parallel to the collector, making them perpendicular to the analyzer's axis z. The top grids, g_1 and g_2, along with the bottom grid, g_6, are kept at the same potential as the satellite structure. This setup aims to prevent disturbances in the surrounding plasma that could arise from potential fluctuations in the adjacent grids g_3, g_4, and g_5. The next two grids, g_3 and g_4, are retarding grids. The positive potential, g, applied to these grids prevents ions moving in the +z direction with energies lower than e2 g from reaching the collector. The retarding potential can range from -2 to +22V, allowing it to suppress ionospheric ions from H^+ to Fe^+. Each grid consists of a network of wires arranged perpendicular to each other, with neighboring parallel wires spaced 0.5 mm apart. The cross-sectional area of each wire is square-shaped, with each side measuring approximately 0.03 mm. The potential depression in the region between the grid wires depends on the grid separation distance, d, the spacing between the wires, a, and the thickness of the wires,δ. Under the conditions where δ/a<<1 and d/a>1, the average potential depression can be expressed in the following form.

$$\varphi^* \approx \varphi_g \left(1 - \frac{ka}{2\pi d^*} \log \left[\frac{a}{\pi\delta} \right]\right) \text{.......(i)}$$

Here, τ represents the leakage parameter of the square grid in comparison to the linear grid. The effective grid separation distance d_{eff} is equal to d/2 when there is a single retarding grid in the configuration, and it becomes d in the configuration with a double grid. For the APR design with τ≈1.72\ and d=3mm, the average potential depression is estimated to be approximately 0.85Vg for the single-grid configuration and approximately 0.92Vg for the double-grid configuration.

A negative potential of -12V is applied to grid g5 with three main purposes: (i) to block photoelectron current from reaching the collector, (ii) to prevent thermal electrons from accessing the collector, and (iii) to minimize the emission of secondary electrons from the collector. Despite these effects, the grid system ensures that the initial energy of the particles reaching the collector remains unchanged.

2. Velocity Direction Analyzer: The ADV analyzer consists of the following components (Fig. 1b): (i) a collector with a radius of 35.5 mm, (ii) an entrance diaphragm with a side length of 30 mm positioned 20 mm above the collector, and (iii) seven parallel grids mounted above the collector. To avoid disturbances in the surrounding plasma caused by potential fluctuations on the grids g2 and g7, the external grid (g1) and the internal grids (g3, g4, g5, and g6) are grounded.

A positive potential of +2V can optionally be applied to grid g2. This potential suppresses ions with z-directional velocities lower than approximately $2 \cdot 10^4 \left(\frac{m_i}{m_{H+}}\right)^{0.5}$ where m_i is the mass of the ion species and m_{H+} is the mass of a hydrogen ion. Under these conditions, all hydrogen ions and most helium ions will be blocked by this grid potential. This assumes:

i. The bulk plasma velocity in the satellite's frame is primarily determined by the satellite's speed, which is aligned with the z-axis and estimated to be around $7.25 \cdot 10^3 ms^{-1}$.
ii. The thermal speed of ions at an altitude of approximately 750 km does not exceed $6 \cdot 10^3 \left(\frac{m_i}{m_{H+}}\right)^{0.5}$

A negative potential of -12V applied to grid g7, located near the collector, prevents the collection of electron and photoelectron currents, ensuring accurate measurements.

The analyzers' observations regarding the ion flows

1. Approximate estimation of the current (magnitude order): Ion flows reaching the analyzer's collector generate a current, which can be approximately calculated as

$$J = eS \, \Sigma_i F_i \text{.............(ii)}$$

where e is the charge, S is the collector area, and Fi represents the ion flux for each species.Here, e represents the elementary charge (e≈1.6×10-19C), S is the area of the analyzer entrance (1.26×10^{-3} m^2 for APR and 0.9×10^{-3} m^2 for ADV), and F_i is the flux of ion species ii. Assuming the plasma is cold, stationary, and composed of only one ion species with a density n_i the ion flux on the collector can be approximated by $F_i \approx n_i v_{sc}$, where v_{sc} is the satellite speed. The characteristic density of the dominant ion species, either oxygen (on the dayside) or hydrogen (on the nightside), at the satellite altitude (~750 km) is estimated to be around $n_i \approx 10^{11} m^{-3}$. Consequently, the expected currents collected are approximately 500nA for the APR and

460nA for the ADV. However, accurate calculations of the ion fluxes and the resulting currents are complicated by several factors.

- The non-zero temperature of the ion population.
- The non-zero bulk velocity of ion species in the Earth's reference frame.
- The limited angular aperture of the analyzer.
- The retarding effect of the grids.
- Ion losses on the grids and the side surfaces of the analyzer.
- The finite value of the satellite potential, among others.

These factors will be discussed in the following sections.

2. Grid permeability: Before an ion reaches the collector, it passes through several grids positioned between the analyzer entrance and the collector. It is assumed that if an ion collides with a grid wire, it is absorbed and does not reach the collector. The number of ions passing through the grid is proportional to the ratio of the open space between the wires to the total area of the entrance diaphragm. Assuming the ion population is cold and its primary velocity component is aligned with the analyzer axis (perpendicular to the grids), the grid transparency is estimated to be $(a-\varepsilon)^2/a^2 \approx 0.884$.

If the analyzer consists of n grids, the input flux will be reduced by a factor of 0.884^n by the time it reaches the collector. This factor is approximately 0.48 for the APR and 0.42 for the ADV analyzers. If the velocity perpendicular to the analyzer axis is about 10% of the parallel velocity, the transparency factors decrease to approximately 0.44 for APR and 0.39 for ADV.

3. Ion concentration distribution: The ion distribution function is assumed to follow a Maxwellian distribution and be isotropic. As a result, in the plasma frame, it can be expressed as:

$$f_i = f_{0i} e^{-\frac{\beta_i^2}{2v_i^2}} \ldots\ldots\ldots\ldots\ldots (iii)$$

where f_{oi} is the maximum ion distribution function, and v_i represents the ion velocity.Here $\beta_i = \sqrt{\frac{m_i}{2kT_i}}$, where m_i and T_i are the ion mass and temperature, respectively, k is the Boltzmann constant ($k=1.38\times10^{-23}$ J/K), and v_i represents the bulk velocity. The quantity f_{oi} can be related to the ion density n_i, since the density is the first moment of the distribution function. In a spherical coordinate system (v,θ,ϕ), it is expressed as:

$$n_i = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \int_0^{\infty} f v^2 dv = \frac{\pi^{3/2}}{\beta^3} f_{oi} \quad \text{and}$$

$$f_{oi} = \frac{\beta^3}{\pi^{3/2}} n_i = \left[\frac{m_i}{2\pi k T_i}\right]^{3/2} n_i.$$

For simplicity in analytical calculations, we assume that the primary component of the ion velocity is aligned with the analyzer axis and is primarily dictated by the satellite's velocity.In the satellite's reference frame, the distribution function can be expressed as follows.

$$F_i = F_{oi} \exp(-\beta_i^2 (v - v_{||})^2)$$

with $v = \{v\cos\theta,\ v\sin\theta\cos\varphi,\ v\sin\theta\sin\varphi\}$ $\{v_{||},0,0\}$, and can be re-written in the form

$$F_i = F_{oi} \exp(-\beta_i^2 v_{||}^2) \exp(-\beta_i^2 v^2 + 2\beta_i^2 \cos\theta\, v_{||} v).$$

Ion flow reaching the APR collector

1. 1-D analytical solution: The ion flux on the APR collector is a key parameter for understanding plasma-analyzer interactions, as it determines the rate at which ions from the plasma environment reach the collector. A one-dimensional (1-D) analytical solution provides a simplified approach by considering only the velocity component parallel to the analyzer's axis, assuming a uniform plasma flow primarily aligned with the satellite's motion. This approximation facilitates initial estimates of ion flux while capturing the essential dynamics of the interaction. Despite its simplicity, the 1-D solution serves as a foundation for interpreting experimental data and lays the groundwork for more complex three-dimensional models.

2. 3-D analytical solution: The three-dimensional (3-D) analytical solution for ion flux on the APR collector provides a more comprehensive approach by accounting for the full velocity distribution of ions in all directions. Unlike the 1-D solution, which only considers the velocity component along the analyzer's axis, the 3-D model incorporates transverse velocity components and angular effects, offering a detailed representation of ion trajectories. This method captures the influence of non-parallel ion motions, finite angular apertures, and other geometrical factors of the analyzer. By providing a more realistic depiction of ion behavior, the 3-D analytical solution enables precise predictions of ion flux, particularly under complex plasma conditions, and enhances the interpretation of experimental data in dynamic environments.

Computational modeling through the Monte Carlo method: There is an effect that is challenging to estimate analytically, specifically the loss of ions on the analyzer's side walls under conditions of a non-zero retarding potential. To address this, a numerical simulation using the Monte Carlo method provides a means to quantify this effect and verify the accuracy of analytical solutions. The core concept of this approach is to simulate a large number, NN, of test particles with velocity distributions matching the expected plasma conditions. The trajectories of these particles are tracked within the analyzer, and the current associated with particles reaching the collector is calculated.

The initial position of each test particle on the first diaphragm (g_1) with cross-sectional area S_1 is assigned randomly. Its velocity components are set as $v_z=v_k+(GV_T)$, $v_x=v_{xo}+(GV_T)$, and $v_y=v_{yo}+(GV_T)$, where (GV_T) is a Gaussian probability function with thermal width V_T and $v_k=\{v_{xo},v_{yo},v_k\}$ represents the bulk velocity of the ion population in the satellite frame. The particle's trajectory is then traced step by step, with each "step" representing either a grid or the collector, both considered as equipotential planes.

Comparison between analytical and computational solutions: Ions that enter the analyzer are partially absorbed by the side structures, meaning not all of them reach the collector. If the plasma is assumed to be cold and moving along the instrument's axis, the ions will be lost only on the structure supporting grid g_2 of the APR (as shown in Fig. 1a). However, all particles that pass through diaphragm g_2 will reach the collector and contribute to the current. In this case, the current on the collector can be calculated using Equation (2) with the flux defined and the entrance area is simply the open section of diaphragm g_2.

Thermal ions will be slowed down by the electric field created by the voltage difference between grids g_2 and g_3. If their velocity component perpendicular to the analyzer axis is not zero, they may collide with the wall structure. The likelihood of such losses depends on the analyzer's geometry and the ratio between the thermal and bulk velocities. Specifically, if the perpendicular displacement of an ion between grids g_2 and g_3 exceeds the difference in radii between the g_3 and g_2 diaphragms, the ion will be lost. However, for the APR geometry and the expected values of thermal and bulk velocities, the displacement of thermal ions typically does not exceed $\frac{21zv_T}{v_k}$ where $v_T = \sqrt{\frac{2kT}{m}}$.

Therefore, the displacement is not significant enough to cause major losses, and most thermal ions will still reach the collector.As a result, the current generated on the collector by the thermal ion population can be estimated using the cold plasma approximation, with the diaphragm area replaced by its effective value, S_{eff}. The effective area of the entrance diaphragm can be determined by considering the ratio of ion fluxes obtained from the 1-D and 3-D solutions.

Conclusion: The primary objective of this study was to present and justify simple analytical methods that can be used to derive ion flows from current measurements. It was demonstrated that, under the expected conditions for the Demeter mission—where the bulk plasma velocity in the satellite frame exceeds the ion thermal velocity—the following conclusions hold:

i. The current-voltage response measured by the APR analyzer is well approximated by the 1-D solution, which accounts for the ion thermal motion perpendicular to the analyzer axis by incorporating an effective area for the entrance diaphragm.
ii. Even a small ion population can be identified from the APR response.
iii. The current ratio measured by the ADV sensor can be fitted using a simple geometrical expression, where the size of the entrance diaphragm is replaced by its effective area, provided that the current on the collector is due to ion flows with thermal velocities greater than half the bulk speed.
iv. By combining APR and ADV measurements, it is possible to reconstruct the arrival angles of ion flows in a multi-species plasma.

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